Portfolio optimization
using the GO-GARCH model:
evidence from Ukrainian Stock Exchange

Abstract. This paper provides an experimental study on optimal portfolio composition. Data on seven stocks, included in Ukrainian Exchange Index, for the period from January to December 2015 are considered. In total, seven big industrial, electric and military companies are selected from the Ukrainian Exchange: Avdiivka Coke Plant, PJSC; Azovstal Iron and Steel Works, PJSC; Raiffeisen Bank Aval, JSC; Centerenergo, PJSC; Donbasenergo, PJSC; Motor Sich, JSC; and Ukrmfta, OPJC. The sample amounts to 226 observations.

The analysis covers descriptive statistics, correlation, and, finally, optimal investment weights, which are calculated using Sharpe ratio. Covariance matrix of returns is estimated by means of generalized orthogonal GARCH model with Gaussian and normal-inverse Gaussian distributions for errors. Selected stocks during the considered period have on average negative rates of returns. At the same time, these stocks in most of cases are positively correlated with each other, leading hence to a fewer room for the efficient diversification. Both Gaussian and normal-inverse Gaussian portfolios preclude that on average investment weights one should be focused mainly on Centerenergo and Motor Sich stocks. Based on these results, the investor should buy 46% of Centerenergo’s stocks and 34% of Motor Sich’s stocks.

Selected stocks during the considered period have on average negative rates of returns. At the same time, these stocks in most of cases are positively correlated with each other, leading hence to a fewer room for the efficient diversification. Despite this, our results denoted that implementation of multivariate GARCH together with normal-inverse Gaussian distribution for errors enables to reduce the portfolio risk substantially. Comparing optimal GO-GARCH portfolios with naïve portfolio with all weights equal and Ukrainian Exchange Index we demonstrate that the former provide smaller portfolio variance and better VaR than naïve portfolio and the Index.

Keywords: Portfolio; GO-GARCH Model; Return; Risk; Optimizations; Stock Exchange; Portfolio

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Оптимизация инвестиционного портфеля с помощью модели GO-GARCH (на материалах Украинской фондовой биржи)

Аннотация. В данной работе проведено эмпирическое исследование процесса формирования оптимального портфеля на основе данных по семи акциям, входящим в индекс Украинской биржи, за период с января по декабрь 2015 года. Анализ включает в себя описательную статистику, корреляцию и расчет оптимальных весов в портфеле, основываясь на применении коэффициента Шарпа. Вариационно-ковариационная матрица доходностей оценивается с помощью обобщённой ортогональной GARCH модели с гауссовым и нормальным-обратным гауссовым распределениями для остатков. Результаты исследования показывают, что применение многомерной GARCH модели с нормальным-обратным гауссовым распределением для остатков позволяет существенно снизить портфельный риск. Сравнивая GO-GARCH портфели с наивным портфелем, в котором веса всех активов равны, и индексом Украинской биржи, мы продемонстрировали, что первые обеспечивают меньшую дисперсию портфеля и меньший VaR, чем наивный портфель и индекс.

Ключевые слова: портфель; модель GO-GARCH; доходность; риск; оптимизация.

1. Introduction
Financial intermediaries are essential participants in the investment process. They play an important role in the investment market, acting as intermediaries in the accumulation and redistribution of temporarily free funds. To perform the mission of capital protection and enhancement, financial intermediaries should constantly improve the efficiency of their activities in the securities market and work on improvement of analytical instruments used in management process.

Risk exposure prevention requires intense diversification of the investment portfolio. Improving the asset allocation efficiency for financial intermediaries through diversification can not only significantly reduce the investment risk, the probability and amount of loss on the stock market, but also create conditions for improved financial results.

2. Brief Literature Review
Theoretical and practical aspects of the portfolio investment through diversification, in particular solving problems related to finding an optimal balance of assets in the portfolio, as well as the calculation of their cost, were studied by many prominent scholars, including several Nobel Prize winners in Economics: Harry Markowitz (Markowitz, 1952) [1], William Sharpe (Sharpe, 1964) [2], James Tobin (Tobin, 1985) [3] and others. They developed a theory of the investment portfolio, conducted a mathematical study of criterion “risk-return” and described the construction of optimal portfolio weights.

The investment portfolio theory is based on the mean-variance efficiency for assets allocation, pioneered by Harry Markowitz (Markowitz, 1952, 1959) [1, 4] and further developed by William Sharpe (Sharpe, 1964) [2]. Moreover, Capital Asset Pricing Model (CAPM) was developed by William Sharpe (Sharpe, 1964) [5], Joan Lintner (Lintner, 1965) [6] and Jan Mossin (Mossin, 1966) [7]. Arbitrage pricing theory was pioneered by Stephen Ross (Ross, 1976) [8].

Philosophy of index investing originated in the early 1950s, when John Bogle, Princeton University graduate, in his Master’s thesis showed that two-thirds of mutual funds provided their shareholders, through the implementation of active investment strategies that existed at that time, with yield which was not greater than if they had just carried out investments in shares of companies following the structure of a generally known stock index. Eventually, in 1976 John Bogle founded the first index fund for individual investors, now called Vanguard 500 Index Fund, which had investments in stocks included in the S&P 500 index by buying securities in amounts correlated to weighting factor of the shares in the index. Other indices primarily used for investment are Dow Jones, Russell, and NASDAQ.

Today scientists actively investigate the issue of rational behavior in the securities market in the process of investment portfolio optimization, study how investors make decisions and how they forecast the price of securities. Overall, among the latest contributions to the subject we would like to note the following scholars.


The optimization was performed by employing a Nondominated Sorting Genetic Algorithm (NSGA-II). Mei, X., DeMiguel, V., Nogales, F. J. (Mei, DeMiguel, Nogales, 2016) [12] analyzed the optimal portfolio policy for a multi-period mean-variance investor facing multiple risky assets in the presence of general transaction costs. For proportional transaction costs, they gave a closed-form expression for a no-trade region, shaped as a multi-dimensional parallelogram, and showed how the optimal portfolio policy can be efficiently computed for many risky assets by solving a single quadratic program. Luo, C., Seco, L., Bill Wu L.-L. (Luo, Seco, Bill, 2015) [13] investigated and compared performances of the optimal portfolio selected by using the Orthogonal GARCH (OGARCH) Model, Markov Switching Model and the Exponentially Weighted Moving Average (EWMA) Model in a fund of hedge funds. Vercher, E., Bermudez D. J. (Vercher, Bermudez, 2015) [14] introduced a cardinality constrained multi-objective optimization problem for generating efficient portfolios within a fuzzy mean-absolute deviation framework. They assumed that the return on a given portfolio was modeled by means of LR-type fuzzy variables, whose credibility distributions collect the contemporary relationships among the returns on individual assets. Using daily returns of the S&P 500 stocks from 2001 to 2011, Mainik, G., Mitov, G., Ruschendorf, L. (Mainik, Mitov, Ruschendorf, 2015) [15] performed a backtesting study of the portfolio optimization strategy based on the Extreme Risk Index (ERI). This method used multivariate extreme value theory to minimize the probability of large portfolio losses.

Despite the presence of a number of studies dedicated to «portfolio optimization», the analysis for the Ukrainian Stock Exchange was not conducted, so we decided to fill this gap.

3. The purpose of the article is to use a mathematical model to minimize the risk for a given portfolio. In this study we examine the portfolio consisted of seven stocks, included in...
Ukrainian Exchange Index (on June 16, 2016 list of the UX Index constituent stocks changed, and now it contains only five stocks [16]). We calculate the optimal weights using Sharpe ratio (Sharpe, 1966) [17]. The estimation of portfolio assets’ conditional covariance is conducted by means of generalized orthogonal GARCH model (Van der Weide, R., 2002) [18] with multivariate normal and normal-inverse Gaussian distributions for errors. Comparing optimal portfolios with naïve portfolio with all weights equal we demonstrate that implementation of multivariate GARCH together with normal-inverse Gaussian distribution for errors enables to reduce the portfolio risk substantially.

4. Methodology

Firstly, we have financial time series of length  \( T \):

\[
x_t = (x_{t,1}, \ldots, x_{t,n}), \quad t = 1, \ldots, T
\]

\( x_t \) are observable returns, which are demeaned by vector autoregression model with intercept:

\[
y_t = x_t - E(x_t | F_{t-1}), \quad E(x_t | F_{t-1}) = \Lambda + \beta x_{t-1}, \quad F_{t-1}.
\]

(2)

where \( \Lambda \) and \( \beta \) are matrices of parameters. The resulted variable \( y_t \) is usually called innovations. Innovations are used in GARCH-type models to estimate volatility \( \Sigma_t \):

\[
y_t = \Sigma_t^{1/2} e_t + F(\theta),
\]

(4)

\[
\Sigma_t = E(y_t y_t' | F_{t-1}).
\]

(5)

Standardized innovations \( z_t \) are distributed according to some known distribution \( F \) with parameter set \( \theta \). The multivariate GARCH models are usually estimated by means of maximum likelihood method with log likelihood function:

\[
L_{\theta} = -\frac{1}{2} \left( \log |\Sigma_t| + y_t' \Sigma_t^{-1} y_t \right).
\]

(6)

where means determinant.

We find optimal weights of the portfolio by maximizing the Sharpe ratio \( Z \):

\[
Z = \frac{\mu - r^f}{\sigma_p},
\]

(7)

where \( \mu \) - portfolio mean return, \( \mu - r^f \) - risk-free rate, \( \sigma_p \) - portfolio standard deviation, \( w \) - portfolio weights, \( \Sigma \) - variance-covariance matrix of returns.

Evidently, the sum of weights \( w \) should be equal to 1. The well-known solution of this problem can be written as follows (see, for example, Zivot, 2011) [19]:

\[
w^* = \frac{\Sigma^{-1} (\mu - r^f \cdot 1)}{1' \Sigma^{-1} (\mu - r^f - 1)}
\]

(8)

where \( 1 \) - vector of ones. Due to the fact that most of the assets demonstrate negative mean return (see Table 1) we allow short selling in our portfolio.

We allow to assume time-varying covariance matrix and model it via GO-GARCH model, which enables us to obtain dynamic conditional covariance and control for autocorrelation and heteroskedasticity in returns (Engle, Kroner, 1995) [20].

\[
\Sigma_t = \Sigma_{t-1} X V_t X' \Sigma_{t-1}
\]

(9)

\( X \) is an \( n \times n \) orthogonalization matrix, \( V_t \) - diagonal matrix with diagonal elements \( \psi_t \), which follow the equation:

\[
v_t = c + D(y_{t-1} \odot y_{t-1}) + KV_{t-1},
\]

(10)

where \( D \) and \( K \) - diagonal matrices of parameters, \( c \) - \( n \times 1 \) vector, \( C \) - element-wise product. To ensure \( \Sigma_t \) matrix to be positive definite, elements of \( D, K \) and \( c \) should be positive.

In our paper we chose multivariate Gaussian and normal-inverse Gaussian distributions (see for example Feller, 2008 [21]) for standardized innovations \( \psi_t \). The first one is a parsimonious and heavily studied distribution with many useful properties.

\[
f_{\psi_t}(x) = (2\pi)^{-n/2} |I|^{-1/2} e^{-\frac{1}{2} x' I^{-1} x},
\]

(11)

The distribution takes into account only two first stochastic moments, represented by location parameter \( \mu \) and covariance matrix \( \Sigma \). But there is a well-known fact from empirical finance, that the empirical distribution of returns tends to be skewed (see, e.g. Harvey, Siddique, 2000) [22]). So along with Gaussian distribution we implement normal-inverse Gaussian distribution for errors (Oigard et al., 2005) [23].

\[
f_{\psi_t}(x) = \frac{\delta}{2\sqrt{n-1/2}} \left[ \frac{\alpha}{\pi q(x)} \right]^{(n-1)/2} \exp(\rho(x) |K_{n-1/2}| a(x)),
\]

(12)

where \( \delta \) controls the heaviness of the tails (smaller values of \( \delta \) implies heavier tails); \( \alpha \) is a vector skewness parameter; \( \beta \) is a scale parameter; \( \mu \) is location parameter; \( \Gamma \) account for correlation between assets.

The flexibility of normal-inverse Gaussian distribution allows to take into account the skewness of returns along with the time-dependent volatility.

5. Empirical results

5.1. Data

Daily data used in this study are downloaded from [24]. The period under consideration lasts from January 6, 2015 till December 30, 2015. Totally, seven firms are selected from the Ukrainian Exchange:

- Avdilvka Coke Plant, PJSC, Common (AVDK),
- Azovstal Iron and Steel Works, PJSC, Common (AZST),
- Raiffeisen Bank Aval, JSC, Common (BVAL),
- Centerenergo, PJSC, Common (CEEN),
- Donbasenergo, PJSC, Common (DOEN),
- Motor Sich, JSC, Common (MSICH), and
- Ukrmatta, OPJC, Common (UNAF).

The sample amounts to 226 observations. Table 1 presents descriptive statistics for the selected stocks’ logarithmic returns.

According to Table 1, UNAF, BVAL and MSICH demonstrate higher average rate of return, and, at the same time, moderate risk, estimated by standard error. On the other hand, stocks with low average returns, such as AVDK, AZST and DOEN have higher risk. CEEN is positioned between these two sub-groups with relatively low average return and small risk.

Based on the fundamental relationship between risk and return, we expect that stocks which demonstrate higher return and lower risk are to be invested with positive weights. As this relation will be stronger, proportions of invested funds should decrease.

Table 2 presents correlation coefficients of returns.

Higher correlation coefficient is found between CEEN and UNAF, i.e. 0.48. In overall, more than 70% of correlation coefficients between returns are positive, what leaves little room for further diversification. As suggested in Damodaran (1996) [25], for the risk free rate we choose the rate of long-term Ukrainian government bonds with expiration on 2026, which is 7.75% [26].
5.2. Portfolio weights

Table 3 contains the average weights of all the assets for two selected probability distributions.

Both Gaussian and normal-inverse Gaussian portfolios preclude that on average investment weights one should be focused mainly on Centerenergo (CEEN) and Motor Sich (MSICH). Based on these results, the investor should buy 46% of Centerenergo’s stocks and 34% of Motor Sich’s stocks.

For illustrative purpose we present kernel estimates of portfolio returns with normal errors and of raw assets’ returns on Figure 1 and Figure 2. We also add a naïve portfolio with equal weights and the returns of UX index on the figures.

![Figure 1: Kernel estimates of returns in comparison with Gaussian, naïve portfolio and UX index](image)

Source: Elaborated by the authors

Obviously, both Gaussian and normal-inverse Gaussian portfolios provide smaller portfolio variance than raw assets and naïve portfolio. Moreover the tails of portfolio distributions are much lighter for the estimated portfolios.

Table 4 presents the estimation of portfolio risk via portfolio returns’ standard deviation and VaR.

Here we choose 95% level of confidence for VaR as suggested in Riskmetrics [27]. Naïve portfolio demonstrates the poorest results. Index portfolio performs slightly better. The use of GO-GARCH allows reducing substantially the risk of portfolio estimated by standard deviation and VaR.

We also compare average returns of GO-GARCH portfolios, naïve portfolio and index in Table 5. The returns are multiplied by 1000 for the sake of convenience. GO-GARCH portfolios provide better returns, than naïve and index portfolios. Moreover, they allow investors to take lower risk (see Table 4, Figure 1 and Figure 2).

![Figure 2: Portfolio risk estimates](image)

Source: Elaborated by the authors

<table>
<thead>
<tr>
<th>Tab. 1: Descriptive statistics</th>
<th>AVDK</th>
<th>AZST</th>
<th>BAVL</th>
<th>CEEN</th>
<th>DOEN</th>
<th>MSICH</th>
<th>UNAF</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean</td>
<td>-0.0022</td>
<td>-0.0026</td>
<td>-0.0010</td>
<td>-0.0021</td>
<td>-0.0031</td>
<td>-0.0010</td>
<td>-0.0007</td>
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<tr>
<td>Standard Error</td>
<td>0.0029</td>
<td>0.0020</td>
<td>0.0014</td>
<td>0.0011</td>
<td>0.0016</td>
<td>0.0009</td>
<td>0.0017</td>
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<tr>
<td>Median</td>
<td>-0.0007</td>
<td>-0.0013</td>
<td>-0.0011</td>
<td>-0.0015</td>
<td>-0.0004</td>
<td>-0.0004</td>
<td>-0.0010</td>
</tr>
<tr>
<td>Mode</td>
<td>0.0000</td>
<td>0.0000</td>
<td>0.0000</td>
<td>0.0000</td>
<td>0.0000</td>
<td>0.0000</td>
<td>0.0000</td>
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<tr>
<td>Standard Deviation</td>
<td>0.0442</td>
<td>0.0314</td>
<td>0.0211</td>
<td>0.0167</td>
<td>0.0250</td>
<td>0.0140</td>
<td>0.0260</td>
</tr>
<tr>
<td>Sample Variance</td>
<td>0.0020</td>
<td>0.0010</td>
<td>0.0004</td>
<td>0.0003</td>
<td>0.0006</td>
<td>0.0002</td>
<td>0.0007</td>
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<tr>
<td>Kurtosis</td>
<td>16.2246</td>
<td>6.6425</td>
<td>0.8107</td>
<td>12.9403</td>
<td>2.5899</td>
<td>4.1995</td>
<td>5.3970</td>
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<td>Skewness</td>
<td>1.5407</td>
<td>-0.7623</td>
<td>0.0001</td>
<td>-1.6326</td>
<td>-0.5769</td>
<td>-0.5876</td>
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<td>Range</td>
<td>0.5010</td>
<td>0.3279</td>
<td>0.1275</td>
<td>0.1756</td>
<td>0.1936</td>
<td>0.1207</td>
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<tr>
<td>Minimum</td>
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<td>-0.1920</td>
<td>-0.0642</td>
<td>-0.1261</td>
<td>-0.1054</td>
<td>-0.0722</td>
<td>-0.0991</td>
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<tr>
<td>Maximum</td>
<td>0.3389</td>
<td>0.1359</td>
<td>0.0633</td>
<td>0.0495</td>
<td>0.0882</td>
<td>0.0485</td>
<td>0.1504</td>
</tr>
<tr>
<td>Sum</td>
<td>-0.5021</td>
<td>-0.6392</td>
<td>-0.2408</td>
<td>-0.5144</td>
<td>-0.7381</td>
<td>-0.2448</td>
<td>-0.1595</td>
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Source: Compiled by the authors

<table>
<thead>
<tr>
<th>Tab. 2: Correlation coefficients</th>
<th>AVDK</th>
<th>AZST</th>
<th>BAVL</th>
<th>CEEN</th>
<th>DOEN</th>
<th>MSICH</th>
<th>UNAF</th>
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</thead>
<tbody>
<tr>
<td>AVDK</td>
<td>1.0000</td>
<td></td>
<td></td>
<td></td>
<td></td>
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<tr>
<td>AZST</td>
<td>-0.0010</td>
<td>1.0000</td>
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<td></td>
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<tr>
<td>BAVL</td>
<td>0.0803</td>
<td>0.0173</td>
<td>1.0000</td>
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<tr>
<td>CEEN</td>
<td>0.3357</td>
<td>0.3382</td>
<td>-0.0344</td>
<td>1.0000</td>
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<tr>
<td>DOEN</td>
<td>-0.0474</td>
<td>0.2131</td>
<td>-0.0558</td>
<td>0.3190</td>
<td>1.0000</td>
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<tr>
<td>MSICH</td>
<td>-0.0800</td>
<td>0.0518</td>
<td>0.0570</td>
<td>0.1575</td>
<td>0.1166</td>
<td>1.0000</td>
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<tr>
<td>UNAF</td>
<td>0.0390</td>
<td>0.3315</td>
<td>-0.0547</td>
<td>0.4798</td>
<td>0.3021</td>
<td>0.1712</td>
<td>1.0000</td>
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</table>

Source: Compiled by the authors

<table>
<thead>
<tr>
<th>Tab. 3: Average weights</th>
<th>AVDK</th>
<th>AZST</th>
<th>BAVL</th>
<th>CEEN</th>
<th>DOEN</th>
<th>MSICH</th>
<th>UNAF</th>
</tr>
</thead>
<tbody>
<tr>
<td>Gaussian</td>
<td>-0.0765</td>
<td>-0.0043</td>
<td>0.1076</td>
<td>0.4608</td>
<td>0.1589</td>
<td>0.3430</td>
<td>0.0106</td>
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<tr>
<td>Normal-inverse Gaussian</td>
<td>-0.0541</td>
<td>0.0142</td>
<td>0.1058</td>
<td>0.4418</td>
<td>0.1543</td>
<td>0.3286</td>
<td>0.0094</td>
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</table>

Source: Compiled by the authors

<table>
<thead>
<tr>
<th>Tab. 4: Portfolio risk estimation</th>
<th>sd</th>
<th>VaR</th>
</tr>
</thead>
<tbody>
<tr>
<td>norm</td>
<td>0.0114</td>
<td>-0.0252</td>
</tr>
<tr>
<td>nig</td>
<td>0.0119</td>
<td>-0.0252</td>
</tr>
<tr>
<td>UX</td>
<td>0.0149</td>
<td>-0.0262</td>
</tr>
<tr>
<td>naïve</td>
<td>0.0153</td>
<td>-0.0274</td>
</tr>
</tbody>
</table>


Source: Elaborated by the authors
MONEY, FINANCES AND CREDIT

Fig. 2: Kernel estimates of returns in comparison with normal-inverse Gaussian, naïve portfolio and UX index
Source: Elaborated by the authors

Tab. 5: Average returns of the portfolios

<table>
<thead>
<tr>
<th>Portfolio</th>
<th>Average return*1000</th>
</tr>
</thead>
<tbody>
<tr>
<td>norm</td>
<td>-0.600</td>
</tr>
<tr>
<td>nig</td>
<td>-0.541</td>
</tr>
<tr>
<td>UX</td>
<td>-1.676</td>
</tr>
<tr>
<td>naive</td>
<td>-0.851</td>
</tr>
</tbody>
</table>

Note: The returns are multiplied by 1000
Source: Elaborated by the authors

References


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